Example Problem:
Proper Design of a Thermowell in Power Piping

Suggested for thermowell design suitability analysis

INTRODUCTION

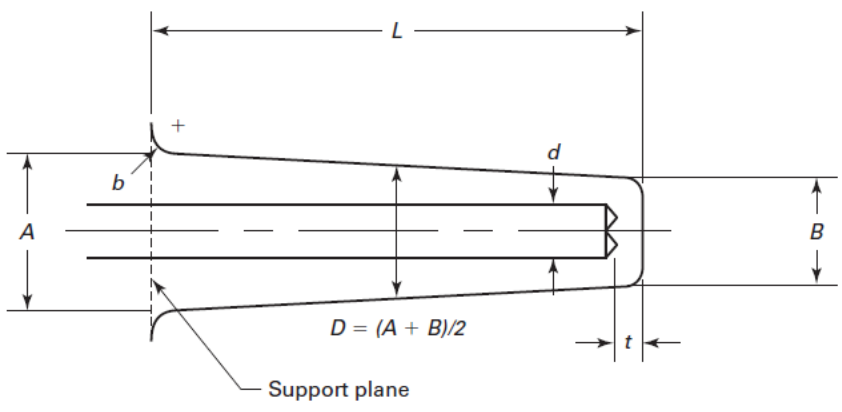
Thermowells are used in measuring the temperature of a moving fluid, where the stream exerts an appreciable force and the sensitive element cannot be placed directly into the medium whose temperature is to be measured. A thermowell is a pressure tight receptacle that is adapted to receive a temperature sensing element and is provided with a variety of process connections (flanges, external threads or a machined shoulder) for tight pressure attachment to a process fitting. The purpose of this example problem is to determine whether or not a thermowell selected for temperature measurement is strong enough to withstand specific application conditions of temperature, pressure, velocity and vibration.

Standards used in the Example Problem:

ASME B31.1-2014, Power Piping
ASME PTC 19.3 TW-2010, Thermowells
ASTM A 105-14, Standard Specification for Carbon Steel Forgings for Piping Applications

THERMOWELL PARAMETERS

Proper design of a thermowell requires that the sensor mounted inside the thermowell attain thermal equilibrium with the process fluid. Thermal modeling of the sensor response is outside the scope of this Standard (refer to the latest version of PTC 19.3 for guidance). Adhering to a few general design rules will optimize the sensor performance within the constraints of the mechanical strength requirements. A high fluid-velocity rating for the thermowell requires a sufficiently high natural frequency for the thermowell and sufficiently low oscillatory stresses. Higher natural frequencies result from decreasing the unsupported length, *L*, increasing the support-plane diameter, *A*, and decreasing the tip diameter, *B*. Lower oscillatory stresses result from decreasing length, *L*, and increasing diameter, *A*. A higher static pressure rating requires increasing the value of tip diameter *B*. In contrast, good thermal performance favors increasing length, *L*, and decreasing diameters, *A* and *B*.



(a) Schematic, Cross-Sectional View of a Thermowell

In addition, if the thermowell is to be evaluated in accordance to ASME PTC 19.3 TW, it is fit for service if it meets these four quantitative criteria:

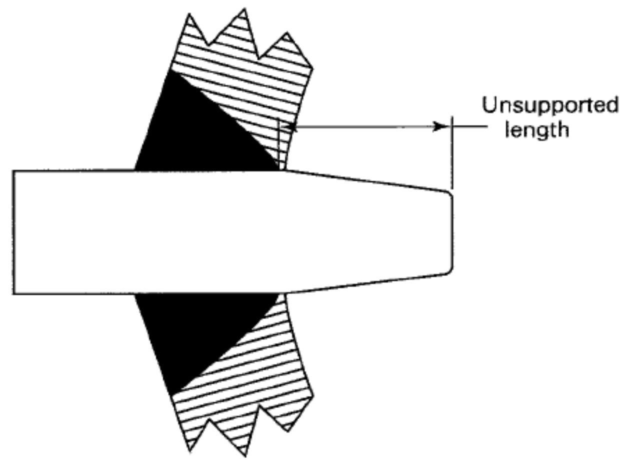
(a) Frequency Limit. The resonance frequency of the thermowell shall be sufficiently high so that destructive oscillations are not excited by the fluid flow.
(b) Dynamic Stress Limit. The maximum primary dynamic stress shall not exceed the allowable fatigue stress limit.
(c) Static Stress Limit. The maximum steady-state stress on the thermowell shall not exceed the allowable stress, as determined by the Von Mises criteria.
(d) Hydrostatic Pressure Limit. The external pressure shall not exceed the pressure ratings of the thermowell tip, shank, and flange.

In addition, the suitability of the thermowell material for the process environment (section 5) shall be considered.

PROBLEM STATEMENT
Consider a thermowell for a steam bypass line, for use under ASME B31.1, *Power Piping*. The designer establishes the process conditions to be:

(a) superheated steam pressure: P = 235 psig
(b) operating temperature: T = 450°F
(c) normal flow condition: V = 295 ft/sec
(d) steam density: ρ= 0.499 lb/ft3
(e) viscosity: μ= 0.0171 cP, or using the conversion factor 1 cP = 6.7197×10^{-4} lb/(ft· sec), μ= 1.149×10^{-5} lb/(ft· sec)

The designer selects a thermowell with a tapered shank, and chooses to include a machined fillet at the root of the shank, which is also the support plane. For this high velocity application, the thermowell is welded directly into the process piping, with the support plane in the heat-affected zone of the weld. The nominal insertion of the thermowell into the process stream is 4 in. The unsupported length, L, exceeds this nominal length due to the possible incomplete penetration of the weld (see image below).



(f) Tapered-Shank Weld-In Thermowell Installed Directly Into Pipe Wall

(a) root diameter: A = 1.5 in.
(b) tip diameter: B = 1.0 in.
(c) fillet radius at base: b = 0.0 in.
(d) bore: d = 0.26 in.
(e) unsupported length: L = 4.06 in.
(f) minimum wall thickness: t = 0.188 in

The designer decides to manufacture the thermowell from ASTM A 105 carbon steel to match the process piping, with the following properties:

(a) from ASME B31.1, Table C-1 (interpolated in temperature), modulus of elasticity at service temperature: E = 27.5×10^6 psi
(b) from ASME B31.1, Table A-1, maximum allowable working stress: S = 19,800 psi
(c) thermowell construction is welded, then machined, so from ASME PTC 19.3 TW Table 6-12.3-1 (Class A, welded), fatigue endurance limit, in the high-cycle limit: S_f = 3,000 psi
(d) from *Metals Handbook Desk Edition* (Davis J.R., CRC Press, 2008), mass density of carbon steel: ρ_m = 0.284 lb/in.^{3}

For the rotational stiffness of the thermowell support, K_M, the designer assumes the thermowell is mounted to a thick-wall pipe (ASME PTC 19.3 TW subsection 6-6) and will use eq. (6-6-5).

For the average density of the temperature sensor, the designer chooses to use the default value found in ASME PTC 19.3 TW, ρ_s = 169 lb/ft^3.

Reynolds and Strouhal Numbers

The Reynolds number is calculated [Eq. 6-4-3] as

$$Re = \frac{VB\rho}{\mu} = \frac{(295\ \text{ft/sec})(1.0\ \text{in})(0.499\ \text{lb/ft}^3)}{(1.149\times 10^{-5}\ \text{lb/(ft}\cdot\text{sec)})(12\ \text{in/ft})} = 1.068\times 10^6$$

For this example Re > 5×10^5, and either eq. (6-4-2) or (6-4-4) gives the Strouhal number N_S = 0.22.

The force coefficients using eq. (6-4-5) are:

$C_D = 1.4$
$C_d = 0.1$
$C_l = 1.0$

Natural Frequency Calculation.

Step 1. Approximate natural frequency [eq. (6-5-1)]:

$$
\begin{aligned}
I &= \pi (D_a^4 - d^4)/64 \\
&= \pi[(1.25 \text{ in})^4 - (0.26 \text{ in})^4]/64 \\
&= 0.1196 \text{ in}^4 \\
m &= \rho_m \pi (D_a^2 - d^2)/4 \\
&= (0.284 \text{ lb/in}^3 \square \pi [(1.25 \text{ in})^2 - (0.26 \text{ in})^2]/4 \\
&= 0.3334 \text{ lb/in}
\end{aligned}
$$

where
$D_a = (1.5 \text{ in} + 1.0 \text{ in})/2 = 1.25 \text{ in}$

Calculate the approximate natural frequency of the thermowell as:

$$
\begin{aligned}
f_a &= \frac{1.875^2}{2\pi}\left(\frac{EI}{m}\right)^{1/2}\frac{1}{L^2} \\
&= \frac{1.875^2}{2\pi}\left(\frac{(27.5\times10^6 \text{ psi})(386.088 \text{ in}\cdot\text{lb/(lbf}\cdot\text{sec}^2))(0.1196 \text{ in}^4)}{0.3334 \text{ lb/in}}\right)^{1/2}\frac{1}{(4.06 \text{ in})^2} \\
&= 2095 \text{ Hz}
\end{aligned}
$$

where
E = the elastic modulus at the operating temperature
$I = \pi (D_a^4 - d^4)/64$, which is the second moment of inertia
L = unsupported length of the thermowell
$m = \rho_m \pi (D_a^2 - d^2)/4$ is the mass per unit length of the thermowell

The conversion factor 386.088 in.-lb = 1 lbf·sec^2 is necessary when E is given in units of pounds per square inch (equivalent to lbf/in.^{2}). (See para. 6-5.3, Step 2 and the Nonmandatory Appendix A.)

Step 2: Use the correlations of subsection 6-5 to correct for deviations from the approximate slender-beam theory:

$$
\begin{aligned}
H_f &= \frac{0.99\left[1+(1-B/A)+(1-B/A)^2\right]}{1+1.1(D_a/L)^{3(1-0.8(d/D_a))}} \\
&= \frac{0.99\left[1+(1-0.6667)+(1-0.6667)^2\right]}{1+1.1(0.3079)^{3(1-0.8(0.2080))}} \\
&= 1.352
\end{aligned}
$$

where
$B/A = (1.0 \text{ in})/(1.5 \text{ in}) = 0.6667$
$D_a/L = (1.25 \text{ in})/(4.06 \text{ in}) = 0.3079$
$d/D_a = (0.26 \text{ in})/(1.25 \text{ in}) = 0.2080$

Step 3: Correct for the fluid mass:

$$
\begin{aligned}
H_{\mathrm{a,f}} &= 1 - \frac{\rho}{2\rho_{\mathrm{m}}} \\
&= 1 - \frac{\left(0.499\ \mathrm{lb/ft}^3\right)}{2\left(0.284\ \mathrm{lb/in}^3\right)\left(1728\ \mathrm{in}^3/\mathrm{ft}^3\right)} \\
&= 0.9995
\end{aligned}
$$

Step 4: Correct for the sensor mass:

$$
\begin{aligned}
H_{\mathrm{a,s}} &= 1 - \frac{\rho_{\mathrm{s}}}{2\rho_{\mathrm{m}}}\left(\frac{1}{\left(D_{\mathrm{a}}/d\right)^2 - 1}\right) \\
&= 1 - \frac{\left(169\ \mathrm{lb/ft}^3\right)}{2\left(0.284\ \mathrm{lb/in}^3\right)\left(1728\ \mathrm{in}^3/\mathrm{ft}^3\right)}\left(\frac{1}{4.808^2 - 1}\right) \\
&= 0.9922
\end{aligned}
$$

where
D_a/d = (1.25 in)/(0.26 in) = 4.808

Step 5: The lowest-order natural frequency of the thermowell with ideal support [eq. (6-5-6)] is given by:

$f_n = H_f\, H_{a,f}\, H_{a,s}\, f_a$
= (1.352)(0.9995)(0.9922)(2095 Hz)
= 2809 Hz .

Step 6: Correct for foundation compliance [eq. (6-6-5)]:

$$
\begin{aligned}
H_{\mathrm{c}} &= 1 - (0.61)\frac{(A/L)}{(1 + 1.5(b/A))^2} \\
&= 1 - (0.61)\frac{(0.3695)}{(1 + 1.5(0))^2} \\
&= 0.7746
\end{aligned}
$$

where
A/L = (1.5 in.)/(4.06 in.) = 0.3695
b/A = (0.0 in.)/(1.5 in.) = 0.0

The in situ natural frequency of the mounted thermowell [eq. (6-6-1)] is given as

$$
\begin{aligned}
f_{\mathrm{n}}^{\mathrm{c}} &= H_{\mathrm{c}}\, f_{\mathrm{n}} \\
&= (0.7746)(2809\ \mathrm{Hz}) \\
&= 2176\ \mathrm{Hz}
\end{aligned}
$$

Scruton Number Calculation
Because the Reynolds number exceeds 10^5, the general frequency limits of para. 6-8.3 apply and no calculation of Scruton number is needed. The calculation is included here as an example. We take a conservative value of 0.0005 for the damping factor, ζ, used in eq. (6-8-1):

$$
\begin{aligned}
N_{\mathrm{Sc}} &= \pi^2 \zeta \left(\rho_{\mathrm{m}}/\rho\right)\left(1-(d/B)^2\right) \\
&= \pi^2 (0.0005)\left(\frac{0.284\ \mathrm{lb/in}^3}{\left(0.499\ \mathrm{lb/ft}^3\right)\left(5.787\times 10^{-4}\ \mathrm{ft}^3/\mathrm{in}^3\right)}\right)\left(1-0.2600^2\right) \\
&= 4.525 \quad .
\end{aligned}
$$

where
d/B = (0.26 in)/(1.0 in) = 0.26

Although N_{Sc} is greater than 2.5, the Reynolds number exceeds 10^5, and the in-line resonance cannot be assumed to be suppressed.

Frequency Limit Calculation.

Step 1: From eq. (6-4-1), the vortex shedding rate with a Strouhal number N_S = 0.22 and at the normal flow condition is

$$
\begin{aligned}
f_S &= \frac{N_S V}{B} \\
&= \frac{(0.22)(295\ \mathrm{ft/sec})(12\ \mathrm{in/ft})}{(1.0\ \mathrm{in})} \\
&= 778.8\ \mathrm{Hz}
\end{aligned}
$$

Step 2: Check that the natural frequency of the mounted thermowell is sufficiently high. In the present example, the thermowell passes the most stringent frequency limit [eq. (6-8-7)]:

$$
f_{\mathrm{S}} < 0.4 f_{\mathrm{n}}^{\mathrm{c}} \quad ,
$$

$$
778.8\ \mathrm{Hz} < 870.2\ \mathrm{Hz} = 0.4(2176\ \mathrm{Hz})
$$

In this case, no calculation of cyclic stress at in-line resonance is needed, because the forced or Strouhal frequency is less than the in-line resonance frequency. However, for the sake of completeness, calculation of this quantity is included in para. 8-1.5.

Cyclic Stress at the In-line Resonance.

Step 1: Use eqs. (6-8-3) and (6-8-4) to establish the flow velocity corresponding to the in-line resonance:

$$
\begin{aligned}
V_{\mathrm{IR}} &= \frac{B f_{\mathrm{n}}^{\mathrm{c}}}{2 N_{\mathrm{S}}} \\
&= \frac{(1.0\ \mathrm{in})(12\ \mathrm{in/ft})^{-1}\left(2176\ \mathrm{Hz}\right)}{2(0.22)} \\
&= 412.0\ \mathrm{ft/sec}
\end{aligned}
$$

Step 2: Evaluate cyclic drag stress at the root. The magnification factor (F'_M) for the drag/in-line resonance is set at 1000 (see paras. 6-8.3, Step 1; and 6-9.2). Begin by evaluating the value of G_{SP}, using eq. (6-10-7):

$$
\begin{aligned}
G_{\mathrm{SP}} &= \frac{16L^2}{3\pi A^2\left(1-(d/A)^4\right)}\left[1+2(B/A)\right] \\
&= \frac{16(4.06\text{ in})^2}{3\pi(1.5\text{ in})^2\left(1-0.1733^2\right)}\left[1+2(0.6667)\right] \\
&= 29.05
\end{aligned}
$$

where
d/A = (0.26 in)/(1.5 in) = 0.1733

From eq. (6-3-3), the force per unit area due to cyclic drag is

$$
\begin{aligned}
P_{\mathrm{d}} &= \frac{1}{2}\rho C_{\mathrm{d}} V_{IR}{}^2 \\
&= \frac{1}{2}\frac{\left(0.499\text{ lb/ft}^3\right)\left(5.787\times10^{-4}\text{ ft}^3/\text{in}^3\right)(0.1)}{(386.088\text{ in}\cdot\text{lb/(lbf}\cdot\text{sec}^2))}\left[(412.0\text{ ft/sec})(12\text{ in/ft})\right]^2 \\
&= 0.9143\text{ psi}
\end{aligned}
$$

where the conversion factor 386.088 in.-lb = 1 lbf·sec^2 is included to give a final answer in units of pounds per square inch (psi).
The cyclic stresses due to cyclic drag [eq. (6-10-6)] at the in-line resonance condition are:

$$
\begin{aligned}
S_{\mathrm{d}} &= G_{\mathrm{SP}} F'_M P_{\mathrm{d}} \\
&= 29.05(1000)(0.9143\text{ psi}) \\
&= 26{,}560\text{ psi}
\end{aligned}
$$

Step 3: Evaluate the stress concentration factor from eq. (6-12-4):

$$K_{\mathrm{t}} = 2.2 \quad .$$

Step 4: Evaluate combined drag and lift stresses, with lift stress set to zero, [eq. (6-12-3)]:

$$
\begin{aligned}
S_{\mathrm{o,max}} &= K_{\mathrm{t}}\left(S_{\mathrm{d}}{}^2 + S_{\mathrm{L}}{}^2\right)^{1/2} = K_{\mathrm{t}} S_{\mathrm{d}} \\
&= 58{,}430\text{ psi}
\end{aligned}
$$

Step 5: Evaluate the temperature de-rating factor from eq. (6-12-6):

$$
\begin{aligned}
F_{\mathrm{T}} &= E(T)/E_{\mathrm{ref}} \\
&= \frac{27.5\times10^6\text{ psi}}{29.3\times10^6\text{ psi}} \\
&= 0.9386
\end{aligned}
$$

The environmental de-rating factor F_E is taken as unity for steam service.

Step 6: Compare the predicted stress with the fatigue stress limit, given by the right hand side of eq. (6-12-5):

$$F_T F_E S_f = (0.9386)(1.0)(3000\,\text{psi})$$
$$= 2816\,\text{psi}$$

The fatigue stress limit, 2816 psi, is less than the combined stress, 58,430 psi. The thermowell would not pass the cyclic stress condition for steady state operation at the in-line resonance, corresponding to a fluid velocity of 412.0 ft/sec, if the vortex shedding frequency f_s had been greater than $0.4 f_n^c$ (see para. 8-1.4, Step 2).

Steady-state Stress at the Design Velocity

Step 1: Evaluate the radial, tangential, and axial stresses due to the external pressure, at the location of maximum stress [eqs. (6-11-1 through (6-11.3)]:

$$S_r = P = 235\,\text{psi} \quad ,$$
$$S_t = P\frac{1+(d/A)^2}{1-(d/A)^2} = (235\,\text{psi})\,\frac{1+(0.1733)^2}{1-(0.1733)^2}$$
$$= 249.6\,\text{psi} \quad ,$$
$$S_a = \frac{P}{1-(d/A)^2} = (235\,\text{psi})\,\frac{1}{1-(0.1733)^2}$$
$$= 242.3\,\text{psi} \quad .$$

Step 2: Evaluate steady-state drag stress at the root. First, evaluate the steady-state drag force per unit area:

$$P_D = \frac{1}{2}\rho C_D V^2$$
$$= \frac{1}{2}\,\frac{\left(0.499\,\text{lb/ft}^3\right)\left(5.787\times10^{-4}\ \text{ft}^3/\text{in}^3\right)(1.4)}{(386.088\ \text{in}\cdot\text{lb/(lbf}\cdot\text{sec}^2))}\left[(295\,\text{ft/sec})(12\ \text{in/ft})\right]^2$$
$$= 6.561\,\text{psi} \quad ,$$

where the conversion factor 386.088 in.-lb = 1 lbf·sec^2 is included to give a final answer in units of pounds per square inch (psi).

Step 3: Evaluate the steady-state stress due to the drag force [eq. (6-10-4)]:

$$S_D = G_{SP} P_D$$
$$= 29.05(6.561\,\text{psi})$$
$$= 190.6\,\text{psi}$$

Step 4: Before using the Von Mises criterion to assess the stress limit at the root, compute the maximum stress given by eq. (6-12-1):

$$S_{max} = S_D + S_a$$
$$= 432.9\,\text{psi}$$

Step 5: Compute the left hand side of the Von Mises criteria, [eq. (6-12-2)]:

$$
\begin{aligned}
\mathrm{LHS} &= \sqrt{\frac{(S_{\mathrm{max}} - S_{\mathrm{r}})^2 + (S_{\mathrm{max}} - S_{\mathrm{t}})^2 + (S_{\mathrm{t}} - S_{\mathrm{r}})^2}{2}} \\
&= 191.0 \text{ psi}
\end{aligned}
$$

Step 6: Compute the stress limit given by the right hand side (RHS) of the Von Mises criteria [eq. (6-12-2)]:

$$
\begin{aligned}
\mathrm{RHS} &= 1.5S \\
&= 1.5(19{,}800 \text{ psi}) \\
&= 29{,}700 \text{ psi}
\end{aligned}
$$

The Von Mises stress, 191.0 psi, does not exceed the stress limit, 29,700 psi, and the thermowell passes the steady-state stress criterion.

Dynamic Stress at the Design Velocity

Step 1: The magnification factor for the lift (transverse) and drag (in-line) resonances are given by eqs. (6-9-1) and (6-9-2), respectively:

$$r = \frac{f_{\mathrm{s}}}{f_{\mathrm{n}}^{\mathrm{c}}} = \frac{778.8 \text{ Hz}}{2176 \text{ Hz}} = 0.3580$$

$$F_{\mathrm{M}} = \frac{1}{1 - r^2} = \frac{1}{1 - 0.3580^2} = 1.147$$

$$r' = \frac{2 f_{\mathrm{s}}}{f_{\mathrm{n}}^{\mathrm{c}}} = \frac{2(778.8 \text{ Hz})}{2176 \text{ Hz}} = 0.7159$$

$$F'_{\mathrm{M}} = \frac{1}{1 - (0.7159)^2} = 2.052$$

Step 2: Evaluate the dynamic drag and lift stresses at the root. Using eq. (6-3-3), the force per unit area due to cyclic drag and lift are:

$$
\begin{aligned}
P_{\mathrm{d}} &= \frac{1}{2}\rho C_{\mathrm{d}} V^2 \\
&= \frac{1}{2}\frac{(0.499 \text{ lb/ft}^3)(5.787 \times 10^{-4} \text{ ft}^3/\text{in}^3)(0.1)}{(386.088 \text{ in} \cdot \text{lb/(lbf} \cdot \text{sec}^2))}\left[(295 \text{ ft/sec})(12 \text{ in/ft})\right]^2 \\
&= 0.4686 \text{ psi}
\end{aligned}
$$

$$
\begin{aligned}
P_{\mathrm{l}} &= \frac{1}{2}\rho C_{\mathrm{l}} V^2 \\
&= \frac{1}{2}\frac{(0.499 \text{ lb/ft}^3)(5.787 \times 10^{-4} \text{ ft}^3/\text{in}^3)(1.0)}{(386.088 \text{ in} \cdot \text{lb/(lbf} \cdot \text{sec}^2))}\left[(295 \text{ ft/sec})(12 \text{ in/ft})\right]^2 \\
&= 4.686 \text{ psi}
\end{aligned}
$$

The cyclic stresses due to drag and lift [eqs. (6-10-5) and (6-10-6)] are:

$$
\begin{aligned}
S_{\mathrm{d}} &= G_{\mathrm{SP}} F_M^{'} P_{\mathrm{d}} \\
&= (29.05)(2.052)(0.4686\ \text{psi}) \\
&= 27.93\ \text{psi} \\
S_{\mathrm{l}} &= G_{\mathrm{SP}} F_{\mathrm{M}} P_{\mathrm{l}} \\
&= (29.05)(1.147)(4.686\ \text{psi}) \\
&= 156.1\ \text{psi}
\end{aligned}
$$

The concentration factor is identical to the value calculate in para. 8-1.5, Step 3, $K_{\mathrm{t}} = 2.2$.

Step 3: Evaluate combined drag and lift stresses, [eq. (6-12-3)]:

$$
\begin{aligned}
S_{\mathrm{o,max}} &= K_{\mathrm{t}} \left(S_{\mathrm{d}}{}^2 + S_{\mathrm{l}}{}^2 \right)^{1/2} \\
&= 2.2 \left((27.93\ \text{psi})^2 + (156.1\ \text{psi})^2 \right)^{1/2} \\
&= 348.9\ \text{psi}
\end{aligned}
$$

Step 4: The temperature de-rating factor is identical to the value calculated in para. 8-1.5, Step 5, $F_{\mathrm{T}} = 0.9386$. The environmental de-rating factor F_{E} is taken as unity for steam service.

Step 5: Compare the predicted stress with the fatigue stress limit, given by the right hand side of eq. (6-12-5):

$$
\begin{aligned}
F_{\mathrm{T}} F_{\mathrm{E}} S_{\mathrm{f}} &= (0.9386)(1.0)(3000\ \text{psi}) \\
&= 2816\ \text{psi}
\end{aligned}
$$

The predicted stress of 348.9 psi is below the fatigue stress limit, and the thermowell passes the dynamic stress criterion.

Pressure Stress

Step1: Compute the external pressure rating for the shank using eq. (6-13-1):

$$
\begin{aligned}
P_{\mathrm{c}} &= 0.66 S \left[\frac{2.167}{2B/(B-d)} - 0.0833 \right] \\
&= 0.66(19{,}800\ \text{psi}) \left[\frac{2.167}{2(1.0\ \text{in})/(1.0\ \text{in} - 0.26\ \text{in})} - 0.0833 \right] \\
&= 9389\ \text{psi}
\end{aligned}
$$

Step 2: Compute the external pressure rating for the tip using eq. (6-13-2):

$$
\begin{aligned}
P_{\mathrm{t}} &= \frac{S}{0.13} \left(\frac{t}{d} \right)^2 \\
&= \frac{19{,}800\ \text{psi}}{0.13} \left(\frac{0.188\ \text{in}}{0.26\ \text{in}} \right)^2 \\
&= 79{,}630\ \text{psi}
\end{aligned}
$$

The pressure rating for the thermowell is the lesser of P_t and P_c, which is 9389 psi in the present case. This rating exceeds the 235 psi operating pressure, and the thermowell passes the external pressure criterion.

The designer was pleased to see that for this set of process conditions, the thermowell was found to be fit for service.